# King Fahd University of Petroleum and Minerals
## Dammam Community College

# MATH 012 – College Algebra II

## Class Test Two

Written Exam, Term 092

May 3, 2010

Solutions & Marking Scheme.

Write your name, ID number and section number.

**Name:**____________________________ **ID #** _______________ **Sec #** _____

**This exam consists of Ten questions.**

**Total _______/45.**

**Time allowed: One hour and fifteen minutes.**

**You must show all necessary steps of your solution.**

**Calculators are not allowed.**

**This test worth 7.5% of the total marks allocated to this course.**

| Question | Marks |
|---|---|
| 1 | /5 |
| 2 | /4 |
| 3 | /6 |
| 4 | /6 |
| 5 | /5 |
| 6 | /4 |
| 7 | /4 |
| 8 | /3 |
| 9 | /4 |
| 10 | /4 |
| Total marks = | /45 |

**1. Find the coordinates of the relative minimum and relative maximum of the graph of $y = \csc\left(\frac{1}{2}x - \frac{\pi}{4}\right)$, on the interval $\left[\frac{\pi}{2}, \frac{9\pi}{2}\right]$.** **[5 marks]**

The corresponding reciprocal function is $y = \sin\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

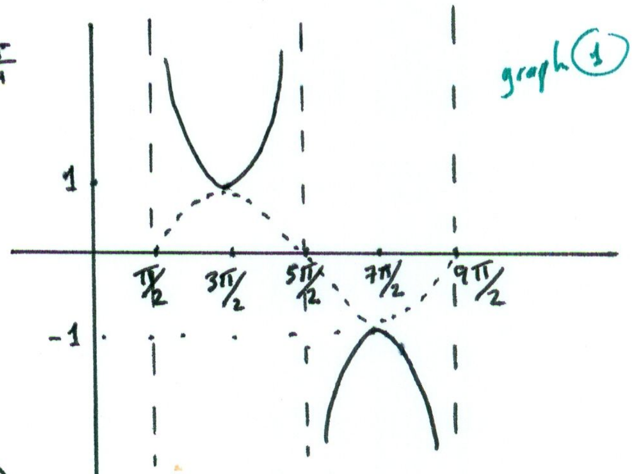
$0 \leqslant \frac{1}{2}x - \frac{\pi}{4} \leqslant 2\pi \qquad + \frac{\pi}{4}$

$\frac{\pi}{4} \leqslant \frac{1}{2}x \leqslant \frac{9\pi}{4}$

$\therefore \frac{\pi}{2} \leqslant x \leqslant \frac{9\pi}{2}$

Period $= \frac{9\pi}{2} - \frac{\pi}{2} = \frac{8\pi}{2} = 4\pi$ (1)

Phase shift $= \frac{\pi}{2}$ (1)




graph (1)

Relative minimum at $\left(\frac{3\pi}{2}, 1\right)$ (A1)

Relative maximum at $\left(\frac{7\pi}{2}, -1\right)$ (A1)

**2. Write the following expression in terms of sine and cosine functions only** **[4 marks]**

$$\frac{1 - \sin^2\theta}{1 + \cot^2\theta} =$$

(1)

$$= \frac{\cos^2\theta}{\csc^2\theta} = \frac{\cos^2\theta}{\frac{1}{\sin^2\theta}}$$

(1)

$$= \sin^2\theta \cdot \cos^2\theta$$

(A2)

**3. Verify that the following trigonometric equation is an identity** **[6 marks]**

$$\frac{(\sec\theta - \tan\theta)^2 + 1}{\sec\theta\csc\theta - \tan\theta\csc\theta} = 2\tan\theta$$

$$LHS = \frac{\sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta + 1}{\csc\theta(\sec\theta - \tan\theta)}$$ 1

1 1

$$= \frac{\sec^2\theta - 2\sec\theta\tan\theta + (\tan^2\theta + 1)}{\csc\theta(\sec\theta - \tan\theta)} = \frac{\sec^2\theta - 2\sec\theta\tan\theta + \sec^2\theta}{\csc\theta(\sec\theta - \tan\theta)}$$

$$= \frac{2\sec^2\theta - 2\sec\theta\tan\theta}{\csc\theta(\sec\theta - \tan\theta)} = \frac{2\sec\theta\,\cancel{(\sec\theta - \tan\theta)}}{\csc\theta\,\cancel{(\sec\theta - \tan\theta)}}$$ 1

1 1

$$= \frac{2\sec\theta}{\csc\theta} = 2\cdot\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} = 2\frac{\sin\theta}{\cos\theta} = 2\tan\theta \equiv R.H.S$$

**4. Find the exact value of** $\tan 165° = \tan(180° - 15°)$ **[6 marks]**

$$= \frac{\tan 180° - \tan 15°}{1 + \tan 180°\tan 15°} = \frac{0 - \tan 15°}{1 + 0.\tan 15°} = -\tan 15°$$ 1

M1

$$\tan 15° = \tan(45° - 30°) = \frac{\tan 45° - \tan 15°}{1 + \tan 45°\tan 15°}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1.\frac{\sqrt{3}}{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$ 1 1

$$= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} = \frac{9 - 3\sqrt{3} - 3\sqrt{3} + 3}{9 - 3}$$

$$= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$$ 1

$$\therefore \tan 165° = -\tan 15° = -(2 - \sqrt{3}) = -2 + \sqrt{3}$$

1

**5. Find the exact value of the following:** **[5 marks]**

$$\frac{\tan\left(\frac{17\pi}{12}\right) - \tan\left(-\frac{\pi}{4}\right)}{1-\tan\left(\frac{5\pi}{12}\right)\tan\left(\frac{\pi}{4}\right)} =$$

$$= \frac{\tan\left(\frac{5\pi}{12}\right) + \tan\left(\frac{\pi}{4}\right)}{1-\tan\left(\frac{5\pi}{12}\right)\tan\left(\frac{\pi}{4}\right)}$$

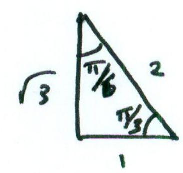
$$= \tan\left(\frac{5\pi}{12}+\frac{\pi}{4}\right) = \tan\left(\frac{8\pi}{12}\right) \quad 1$$

$$= \tan\left(\frac{2\pi}{3}\right) = \tan\left(\pi - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{3}\right) \quad 1$$

$$= -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3} \quad 1$$

$$\tan\left(\frac{17\pi}{12}\right) = \tan\left(\pi + \frac{5\pi}{12}\right) = \tan\frac{5\pi}{12} \quad 1$$

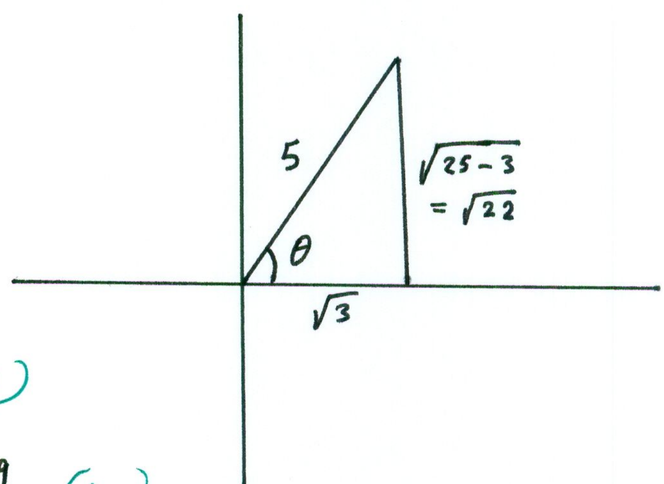
$$\tan\left(-\frac{\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) \quad 1$$




**6. Given** $\cos\theta = \frac{\sqrt{3}}{5}$ **and** $\sin\theta > 0$**, then find the value of** $\cos 2\theta$. **[4 marks]**

$\Downarrow$ Q I, IV $\qquad$ $\Downarrow$ Q I, II

$\therefore\ \theta$ in Q I $\quad$ 1

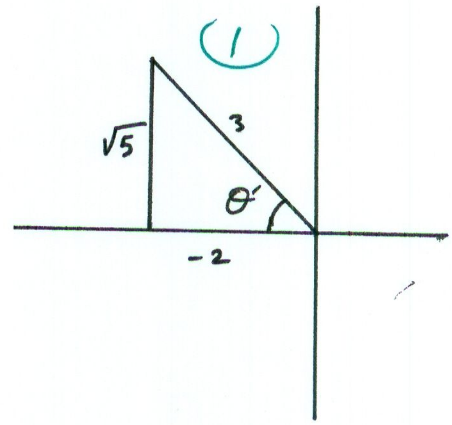



$$\cos 2\theta = \cos^2\theta - \sin^2\theta \quad \text{M1}$$

$$= \left(\frac{\sqrt{3}}{5}\right)^2 - \left(\frac{\sqrt{22}}{5}\right)^2 \quad 1$$

$$= \frac{3}{25} - \frac{22}{25} = -\frac{19}{25} \quad \text{A1}$$

**7.** **Given $\tan\theta = -\frac{\sqrt{5}}{2}$ with $90° < \theta < 180°$, then find the value of $\cot\frac{\theta}{2}$.** **[4 marks]**




$$\cot\frac{\theta}{2} = \frac{1}{\tan\frac{\theta}{2}} = \frac{1+\cos\theta}{\sin\theta} \quad (1)$$

$$= \frac{1 + \left(\frac{-2}{3}\right)}{\frac{\sqrt{5}}{3}} \quad (1)$$

$$= \frac{\frac{3-2}{3}}{\frac{\sqrt{5}}{3}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{5}}{5} \quad (1)$$

**8.** **Given $\cos 2\theta = \frac{1}{2}$ and $\theta$ terminates in quadrant II, Find $\cos\theta =$** **[3 marks]**

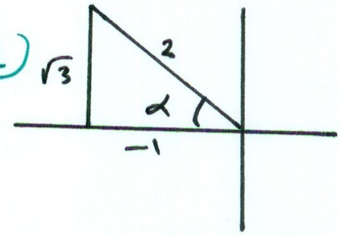
$$\cos\theta = -\sqrt{\frac{1+\cos 2\theta}{2}} \quad (M1)$$ , $-$ve since $\theta$ in QII.

$$= -\sqrt{\frac{1+\frac{1}{2}}{2}} \quad (M1)$$

$$= -\sqrt{\frac{\frac{3}{2}}{2}} = -\sqrt{\frac{3}{4}} = \frac{-\sqrt{3}}{2} \quad (1)$$

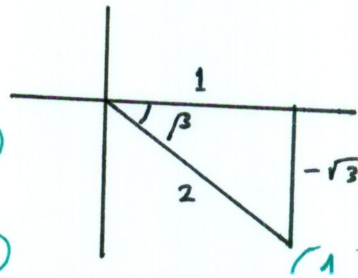
**9. Find the exact value of this expression:** **[4 marks]**

$$\sin\left[\cos^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}(-\sqrt{3})\right] =$$




Let $\alpha = \cos^{-1}\left(-\frac{1}{2}\right)$, $\alpha$ in QII

$\beta = \tan^{-1}(-\sqrt{3})$, $\beta$ in QIV




$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{-1}{2} \cdot \frac{-\sqrt{3}}{2}$ (1)

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$ (1)

**10. Find the exact value of the following:**

a) $\sin^{-1}\left[\sin\left(\frac{15\pi}{4}\right)\right] = \sin^{-1}\left[\sin\left(4\pi - \frac{\pi}{4}\right)\right]$ (M1) **[2 marks]**

$= \sin^{-1}\left[\sin\left(-\frac{\pi}{4}\right)\right] = -\frac{\pi}{4}$ (A1)

b) $\cos^{-1}\left[\cos\left(\frac{5\pi}{4}\right)\right] = \cos^{-1}\left[\cos\left(2\pi - \frac{3\pi}{4}\right)\right]$ **[2 marks]**

$= \cos^{-1}\left[\cos\left(-\frac{3\pi}{4}\right)\right]$ (M1)

$= \cos^{-1}\left[\cos\left(\frac{3\pi}{4}\right)\right]$

$= \frac{3\pi}{4}$ (A1)