King Fahd University of Petroleum and Minerals

Dammam Community College

Term 161

Code A

PREPARATORY YEAR - Math 012

Class Test 2

Dec. 08, 2016

Marking Scheme

Time allowed: 60 Minutes

Name: Marking Scheme ID. ______ Sec # (Key)

Read the following instructions:

1. This test consists of eight questions.
2. You must show all necessary steps of your solution to earn credit.
3. The use of calculators is not allowed.
4. This test worth 8% of the total marks allocated to this course.

Question	Marks
1.	/4
2.	/4
3.	/4
4.	/4
5.	/4
6.	/4
7.	/4
8.	/4
Total Marks	/32

Question 1: Graph: $y = 2\sin\left(x - \frac{\pi}{3}\right)$ over one period. (4 Points)

$0 \leq x - \frac{\pi}{3} \leq 2\pi$ Over one Period

$\frac{\pi}{3} \leq x \leq \frac{7\pi}{3}$

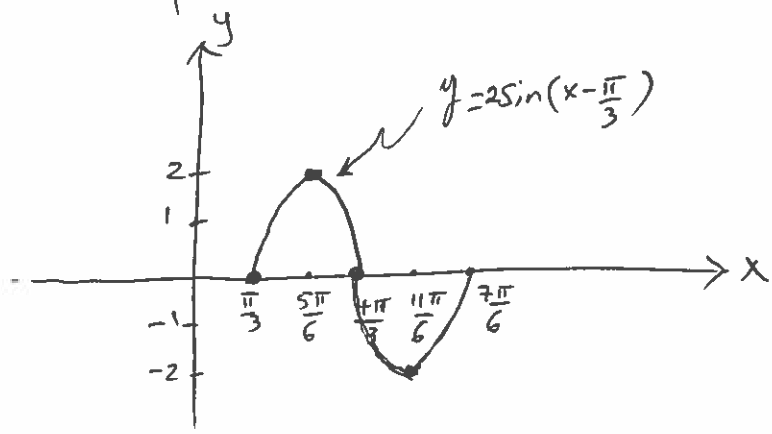
1

$\left[\frac{\pi}{3}, \frac{7\pi}{3}\right]$ divided into four equal parts:

$\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}, \frac{7\pi}{3}$

1.5

x	$\pi/3$	$\frac{5\pi}{6}$	$\frac{4\pi}{3}$	$\frac{11\pi}{6}$	$\frac{7\pi}{3}$
$x - \pi/3$	0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
$2\sin\left(x - \frac{\pi}{3}\right)$	0	2	0	-2	0



1.5

Question 2: Given the function: $y = 3\sec\left(\frac{x}{4} - \frac{\pi}{2}\right)$, find the *period* and the *phase shift*.

(4 Points)

Period $= P = \frac{2\pi}{b} = \frac{2\pi}{1/4} = 8\pi$ — 2

Phase shift : $\frac{x}{4} - \frac{\pi}{2} = 0 \Rightarrow \frac{x}{4} = \frac{\pi}{2} \Rightarrow x = 4\left(\frac{\pi}{2}\right)$

$x = 2\pi$ to the right — 2

Question 3: If $\tan\theta = \frac{-7}{3}$ *and* θ is in quadrant II , find $\sin\theta$. (4 points)

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(-\frac{7}{3}\right)^2 + 1 = \sec^2\theta$$

$$\frac{49}{9} + \frac{9}{9} = \sec^2\theta$$

$$\frac{58}{9} = \sec^2\theta \Rightarrow \sec\theta = -\frac{\sqrt{58}}{3} \quad \text{, since } \theta \text{ in } Q\,\text{II} \quad 1$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{-3}{\sqrt{58}} = \frac{-3\sqrt{58}}{58} \quad 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \sin\theta = \tan\theta\cos\theta$$

$$= \left(-\frac{7}{3}\right)\left(\frac{-3\sqrt{58}}{58}\right) \quad 1$$

$$= \frac{7\sqrt{58}}{58} \quad 1$$

Question 4: Verify that the following equation is an identity:

$$\frac{1+\sin\theta}{\cos\theta}=\frac{\cos\theta}{1-\sin\theta}$$ (4 Points)

L.H.S. $\frac{1+\sin\theta}{\cos\theta} = \frac{(1+\sin\theta)}{\cos\theta} \cdot \frac{(1-\sin\theta)}{(1-\sin\theta)}$ 2

$= \frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}$ 1

$= \frac{\cancel{\cos^2\theta}}{\cancel{\cos\theta}(1-\sin\theta)}$ 1

$= \frac{\cos\theta}{1-\sin\theta}$ R.H.S.

Question 5: Suppose that A and B are angles in standard position, with $\sin A = \frac{4}{5}$,

$\frac{\pi}{2} < A < \pi, and \cos B = \frac{-5}{13}, \pi < B < \frac{3\pi}{2}. Find \tan(A+B)$. (4 points)

$\sin^2 A + \cos^2 A = 1$

$\cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}$

$\cos A = -\frac{3}{5}$, A in Q II. — 1

$\sin^2 B + \cos^2 B = 1$

$\sin^2 B = 1 - \cos^2 B = 1 - \left(\frac{-5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$

$\sin B = -\frac{12}{13}$, B in Q III — 1

$\tan A = \frac{\sin A}{\cos A} = \frac{4/5}{-3/5} = -4/3$ — 0.5

$\tan B = \frac{\sin B}{\cos B} = \frac{-12/13}{-5/13} = 12/5$ — 0.5

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{(-4/3) + (12/5)}{1 - (-4/3)(12/5)} = \frac{16/15}{1 + \frac{48}{15}} = \frac{16/15}{63/15} = \frac{16}{63}$ — 1

Question 6: $Given: \cos\theta = \frac{3}{5}\, and \sin\theta < 0,\, find\ \tan 2\theta.$ (4 points)

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

To find $\tan\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - (3/5)^2$$
$$= 16/25$$
$$\sin\theta = -4/5 \quad — 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-4/5}{3/5} = -4/3 \quad — 1$$

1

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(-4/3)}{1-(-4/3)^2} = \frac{-8/3}{1-\frac{16}{9}} = \frac{-8/3}{-7/9} = \frac{24}{7} \quad — 1$$

Question 7: *Evaluate* : $\cos\left(\arctan\sqrt{3}+\arcsin\frac{1}{3}\right)$. (4 points)

Let $A = \arctan\sqrt{3} \Rightarrow \tan A = \sqrt{3}$

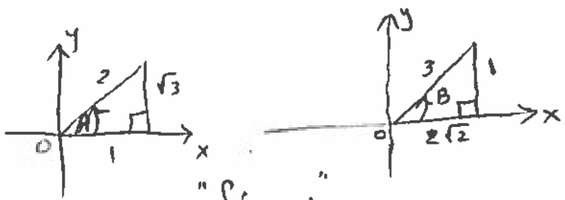
$B = \arcsin\frac{1}{3} \Rightarrow \sin B = 1/3$ — 2

$\cos\left(\arctan\sqrt{3} + \arcsin\frac{1}{3}\right) = \cos(A+B)$

$= \cos A \cos B - \sin A \sin B$ 1

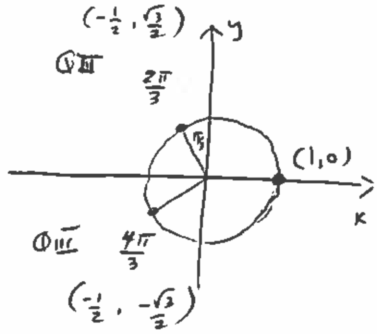
from figure "1" $\longrightarrow$ $= \frac{1}{2} \cdot \frac{2\sqrt{2}}{3} - \frac{\sqrt{3}}{2} \cdot 1/3$

$= \frac{\sqrt{2}}{3} - \frac{\sqrt{3}}{6} = \frac{2\sqrt{2}-\sqrt{3}}{6}$ 1



"figure 1"

Question 8: *Solve*: $\cos 2\theta = \cos\theta$ over the interval $[0, 2\pi)$. (4 points)



$$\cos 2\theta = \cos\theta$$

$$\left.\begin{array}{l} 2\cos^2\theta - 1 = \cos\theta \\ 2\cos^2\theta - \cos\theta - 1 = 0 \end{array}\right\} 1$$

$$\left.\begin{array}{l} (2\cos\theta + 1)(\cos\theta - 1) = 0 \\ 2\cos\theta + 1 = 0 \quad \text{or} \quad \cos\theta - 1 = 0 \\ \cos\theta = -1/2 \qquad \cos\theta = 1 \end{array}\right\} 1$$

$$\left.\begin{array}{l} \text{Q II} \qquad \text{Q III} \qquad \theta = 0 \\ \theta = \frac{2\pi}{3} \text{ or } \theta = \frac{4\pi}{3} \end{array}\right\} 1$$

The Solution: $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, 0$ 1

or $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$