King Fahd University of Petroleum and Minerals

Dammam Community College

Term 161

Code A

PREPARATORY YEAR - Math 012

KEY

marking scheme

Class Test 1
Oct. 20, 2016

Time allowed: 55 Minutes

Name: _____ **ID.** _____ **Sec #** _____

Read the following instructions:

1. This test consists of seven questions.
2. You must show all necessary steps of your solution to earn credit.
3. The use of calculators is not allowed.
4. This test worth 8% of the total marks allocated to this course.

Question	Marks
1.	/5
2.	/5
3.	/4
4.	/6
5.	/4
6.	/4
7.	/4
Total Marks	/32

Question 1: If $f(x)=\frac{1}{x-3}$, $x\neq 3$. Find $f^{-1}(x)$ and its range.

(5 Points)

$$f(x)=\frac{1}{x-3}\ ,\quad x\neq 3$$

$$y=\frac{1}{x-3}$$ (1)

$$x=\frac{1}{y-3}\ ,\quad y\neq 3$$ (1)

$$x(y-3)=1 \Rightarrow xy-3x=1$$

$$xy=3x+1$$

$$y=\frac{3x+1}{x}$$ (1)

$$f^{-1}(x)=\frac{3x+1}{x}\ ,\quad x\neq 0$$ (1)

Range of $f^{-1}(x)$ = Domain of $f(x)=(-\infty,3)\cup(3,+\infty)$ (1)

Question 2: Solve: $\left(\sqrt[3]{5}\right)^{-x} = \left(\frac{1}{5}\right)^{x+\frac{1}{2}}$. (5 Points)

$\left(5^{\frac{1}{3}}\right)^{-x} = \left(5^{-1}\right)^{x+\frac{1}{2}}$ (1)

$5^{-\frac{1}{3}x} = 5^{-x-\frac{1}{2}}$ (1)

$-\frac{1}{3}x = -x - \frac{1}{2}$ (1)

$x - \frac{1}{3}x = -\frac{1}{2}$

$\frac{2}{3}x = -\frac{1}{2}$ (1)

$x = -3/4$ (1)

Question 3: Solve: $x = \log_4 \sqrt[3]{16}$ (4 Points)

$4^x = \sqrt[3]{16}$ (1)

$4^x = (16)^{1/3}$ (1)

$4^x = (4^2)^{1/3}$ } (1)

$4^x = 4^{2/3}$

$x = 2/3$ (1)

Question 4: Show that:

$$3\log_2 t - \frac{1}{3}\log_4 t^{12} + \frac{1}{2}\log_{\frac{1}{2}} t^6 = \log_2\left(\frac{1}{t^2}\right), \text{ for } t > 0. \qquad \text{(6 Points)}$$

$\log_2 t^3 - \log_4 (t^{12})^{1/3} + \log_{\frac{1}{2}} (t^6)^{1/2} = R.H.S'$

$\log_2 t^3 - \log_4 t^4 + \log_{\frac{1}{2}} t^3 = RHS$ } (1)

$\log_4 t^4 = \frac{\log_{10} t^4}{\log_{10} 4} = \frac{4\log t}{2\log 2} = \frac{2\log t}{\log 2} = \frac{\log t^2}{\log 2} = \boxed{\log_2 t^2}$ (2)

$\log_{\frac{1}{2}} t^3 = \frac{\log_{10} t^3}{\log_{10} \frac{1}{2}} = \frac{\log t^3}{\log 2^{-1}} = \frac{\log t^3}{-\log 2} = \boxed{-\log_2 t^3}$ (2)

$\therefore$ L.H.S.: $\log_2 t^3 - \log_2 t^2 - \log_2 t^3 = R.H.S'$

$-\log_2 t^2 = R.H.S'.$

$\log_2 t^{-2} = R.H.S'.$

$\log_2\left(\frac{1}{t^2}\right) = R.H.S' = \log_2\left(\frac{1}{t^2}\right)$ (1)

Q.E.D.

Question 5: Find the solution set of the equation:

$$\log_5(x+2)+\log_5(x-2)=1$$

(4 Points)

$\log_5(x+2)(x-2)=1 \Rightarrow \log_5(x^2-4)=1$ (1)

$x^2-4=5^1$ } (1)

$x^2=9$

$x=\pm 3$, check $x\neq -3$ (1)

Solution Set: $\{3\}$. (1)

Question 6: If α is the complementary of $39^\circ 50'$ and β is the supplementary angle of $14^\circ 20''$. (4 Points)

Find $2\alpha + \beta$.

$\alpha: 90^\circ - 39^\circ 50' :$

$$\begin{array}{r} 89^\circ\ 60' \\ -\ 39^\circ\ 50' \\ \hline 50^\circ\ 10' \end{array}$$

(1)

$\beta: 180^\circ - 14^\circ 20'' :$

$$\begin{array}{r} 179^\circ\ 59'\ 60'' \\ 14^\circ\ 00'\ 20'' \\ \hline 165^\circ\ 59'\ 40'' \end{array}$$

(1)

$2\alpha = 2(50^\circ 10') =$

$$\begin{array}{r} 100^\circ\ 20' \\ +\ 165^\circ\ 59'\ 40'' \\ \hline 265^\circ\ 79'\ 40'' \end{array}$$

$\leftarrow \beta$ (1)

$266^\circ\ 19'\ 40''$ (1)

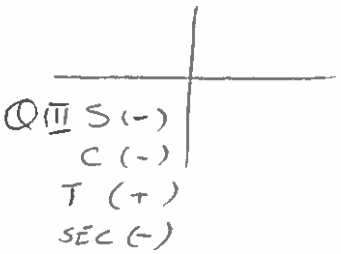
Question 7: Given that $\tan\theta = \frac{3}{4}$ where θ is in quadrant III. Find the value of $\sin\theta$.

(4 Points)

$\tan^2\theta + 1 = \sec^2\theta$

$\left(\frac{3}{4}\right)^2 + 1 = \sec^2\theta \Rightarrow \frac{9}{16} + 1 = \sec^2\theta$

$\frac{25}{16} = \sec^2\theta$ } (1)



$\sec\theta = \pm\sqrt{\frac{25}{16}} = \pm\frac{5}{4}$

$\sec\theta < 0$ in QIII $\therefore$ $\sec\theta = -5/4$ (1/2)

$\therefore \cos\theta = \frac{1}{\sec\theta} = -\frac{4}{5} < 0$ in QIII (1)

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(-\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}$

$\sin\theta = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$ } (1)

$\sin\theta = -3/5 < 0$ in QIII (1/2)