

King Fahd University of Petroleum and Minerals
Dammam Community College



MATH 012
Class Test 2

CODE B

Term 171
November 21, 2017

Name: Marking scheme ID # _____ Sec # _____

This test consists of 6 questions.

Time allowed. 60 minutes.

You must show all necessary steps of your solution.

Calculators are not allowed.

This test worth 8% of the total marks allocated to this course.

Question	Points
Q#1	/4
Q#2	/4
Q#3	/4
Q#4	/4
Q#5	/4
Q#6	/4
Total points =	/24

Question 1: Given that $\sin \alpha = \frac{1}{\sqrt{2}}$, α is in the first quadrant, and $\cos \beta = \frac{\sqrt{3}}{2}$, β is in the fourth quadrant, then find $\sin(\alpha + \beta)$

$$\cos \alpha = +\sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}} \quad (+1)$$

$$\sin \beta = -\sqrt{1 - \cos^2 \beta} = -\sqrt{1 - \frac{3}{4}} = -\sqrt{\frac{1}{4}} = -\frac{1}{2} \quad (+1)$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad (+1)$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \quad (+1)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3} - 1}{2\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4} \quad (+1)$$

Question 2: Find all x-intercepts of the graph of $y = 3 \tan\left(\frac{x}{2} - \pi\right)$, over $-3\pi \leq x \leq 3\pi$.

x-intercepts $\frac{x}{2} - \pi = n\pi \quad (+1.5)$

$$\frac{x}{2} = n\pi + \pi$$

$$(+1) \quad x = 2n\pi + 2\pi, \quad n = \dots, \pm 1, \pm 2, \dots$$

$$n = 0 \Rightarrow x = 2\pi$$

$$n = -1 \Rightarrow x = 2(-1)\pi + 2\pi = 0$$

$$n = -2 \Rightarrow x = 2(-2)\pi + 2\pi = -2\pi$$

} $\left(\frac{1}{2}\right)$ for each

Question 3:

- a) If $A = 3\sin^2(2x) + 3\cos^2(2x)$ and $B = 5\tan^2(x) - 5\sec^2(x)$.
Find $A - B$.

$$A = 3(\sin^2(2x) + \cos^2(2x)) = 3(1) = 3 \quad \left(+\frac{1}{2}\right)$$

$$B = 5(\tan^2(x) - \sec^2(x)) \quad \left(+\frac{1}{2}\right)$$

$$= 5(\tan^2(x) - (\tan^2(x) + 1)) \quad \left(+\frac{1}{2}\right)$$

$$= 5(-1) = -5$$

$$A - B = 3 - (-5) = 8 \quad \left(+\frac{1}{2}\right)$$

- b) Verify $\tan^2 x (\cot^2 x + 1) = \sec^2 x$

$$\text{L.H.S} = \tan^2(x) (\cot^2 x) \quad \left(+\frac{1}{2}\right)$$

$$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} \quad \left(+\frac{1}{2}\right)$$

$$= \frac{1}{\cos^2 x} \quad \left(+\frac{1}{2}\right)$$

$$= \sec^2 x = \text{R.H.S} \quad \left(+\frac{1}{2}\right)$$

Question 4: Find the exact value of $\sqrt{3} \sin\left(\frac{17\pi}{3}\right) - \sqrt{3} \tan\left(\frac{25\pi}{6}\right)$

Coterminal of $\frac{17\pi}{3} = \frac{17\pi}{3} - \frac{12\pi}{3} = \frac{5\pi}{3}$ (+1)
 reference angle = $2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$

$\sin\left(\frac{17\pi}{3}\right) = \sin\left(\frac{5\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ (+1)

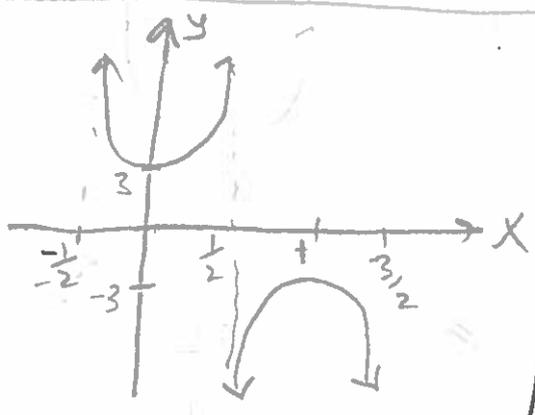
Coterminal of $\frac{25\pi}{6} = \frac{25\pi}{6} - 4\pi = \frac{\pi}{6}$ (+1)

$\tan\left(\frac{25\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$ (+1/2)

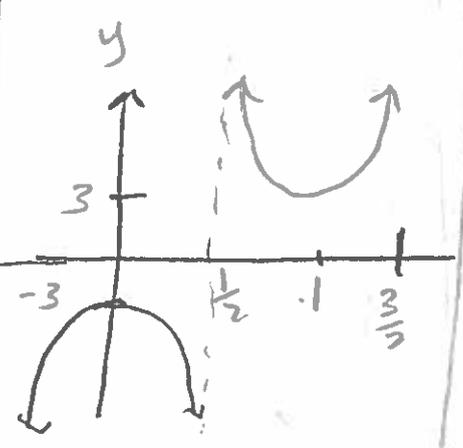
$\sqrt{3} \sin\left(\frac{17\pi}{3}\right) - \sqrt{3} \tan\left(\frac{25\pi}{6}\right) = -\sqrt{3} \cdot \frac{\sqrt{3}}{2} - \sqrt{3} \cdot \frac{1}{\sqrt{3}} = -\frac{3}{2} - 1 = -\frac{5}{2}$ (+1/2)

Question 5: Graph $y = -3 \sec(\pi x - 2\pi) + 1$ over one period

$y = -3 \sec(\pi(x-2))$ (+1/2) y period = $\frac{2\pi}{\pi} = 2$ (+1/2)

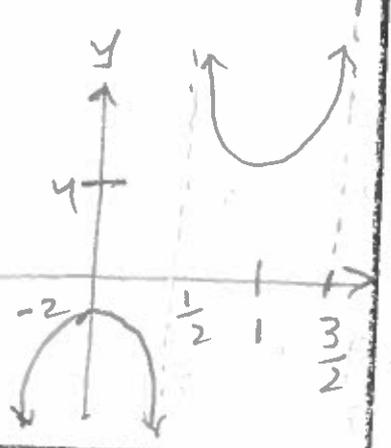


$y = 3 \sec(\pi x)$



$y = -3 \sec(\pi x)$

$y = -3 \sec(\pi x - 2\pi) + 1$



Question 6: For the function $f(x) = -\cos\left(2\pi x - \frac{\pi}{2}\right) - 3$, A is the amplitude, P is period, M is its maximum value and m is its minimum value. Find $A + P + M + m$.

$$A = |-1| = 1 \quad (+1)$$

$$P = \frac{2\pi}{2\pi} = 1 \quad (+1)$$

$$d = -3$$

$$M = A + d = 1 - 3 = -2 \quad (+\frac{1}{2})$$

$$m = -A + d = -1 - 3 = -4 \quad (+\frac{1}{2})$$

$$A + P + M + m = 1 + 1 - 2 - 4 = -4 \quad (+1)$$